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Reprinted from *Financial Management*
Autumn 1982 Issue

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■ The recent period of unparalleled high interest rates has caused many institutions and individuals to focus attention on short term debt securities as potential investments. This paper will demonstrate that the present method for calculating yields on most short term securities makes comparison of different securities difficult, and contains both a theoretical inconsistency and a serious bias — a bias which varies with the maturity of the instrument, and which becomes more severe at higher yields. The inconsistency and the bias have serious implications for investors, who may be making investment decisions based on incorrect estimates of relative yields, and for researchers, who may be using incorrect data in their analyses.

Present Practice

Treasury bills, bankers acceptances, finance paper, and commercial paper are sold on a bank discount basis. Bank discount yields understate effective annual yields because they are calculated using a 360 day year, rather than a 365 (or 366) day year, and using maturity value, rather than purchase price, in the yield calculation.[1] As long as all securities under consideration are quoted at bank discount, comparison of securities is not difficult, although all yields are understated. However, as soon as non-discounted instruments and coupon issues are considered as alternatives, the potential for error becomes great. To alleviate this comparison problem, many data sources report the coupon equivalent (or bond equivalent) yield as well as the bank discount yield. Appendix A demonstrates the conversion process, which involves adjusting the yield to reflect the purchase price and the actual number of days in a year; the result is the annualized yield assum-

The authors appreciate the extremely useful comments of two anonymous reviewers.

[1]Negotiable CD rates and the London interbank offering rate (LIBOR) are not quoted at bank discount, but are calculated using a 360 day year.

ing simple interest.

Bank discount issues with more than one-half year to maturity have a disadvantage relative to coupon issues of comparable maturity because coupon issues pay interest every six months, while bank discount issues pay interest only at maturity. Accordingly, calculation of coupon equivalent yields for such longer term discount issues requires a second step to adjust the simple yield downward for this disadvantage. This process is demonstrated in Appendix B.

The Problem

The coupon equivalent conversion formula for bank discount issues with maturities in excess of one-half year explicitly recognizes the compounding effect that is possible if coupon issues are purchased but is foregone if longer-term bank discount issues are chosen instead. But it is inconsistent to acknowledge the compounding disadvantage inherent in issues that pay interest less frequently than do coupon issues, and ignore the compounding advantage possible with issues that pay interest more frequently than do coupon issues. Yet this is precisely the present practice, as shown in Appendices A and B.

The remaining sections of this paper will demonstrate that this practice induces a serious computational bias into reported yields, examine the magnitude of the bias, and examine the implications of the bias. A 365 day year will be used in all calculations, and one-half year will be considered one period. As is standard practice for coupon issues, the annualized yield will be calculated as twice the one period yield, rather than compounding the one period rate. The formula for direct computation of the compound yield is shown in Appendix C. And, for compounding purposes, reinvestment will be assumed to be at the initial rate of return.

It should be noted that any return or yield computation is merely a standardization convention that is employed because it is useful. Annual, semi-annual, monthly, daily, or continuous compounding, or even simple interest, are no less "correct" than the formula presented in Appendix C. However, this formula does allow for better comparisons between investment alternatives with differing maturities, and hence is more useful in the context of this paper.[2]

The Bias

The computational bias induced by present practices results from using simple interest to approximate compound interest for securities with maturities other than one-half year. Let:

P = the purchase price of a security with par value of 100;

$i = \dfrac{100 - P}{P}$, the simple interest yield earned on a security, unadjusted for time;

t = the number of times i can be earned in one-half year (*i.e.*, 182.5/the number of days to maturity of the security).

The accumulation function $(1+i)^t$ for nonintegral t can be expanded using the binomial expansion [4]:

$$(1+i)^t = 1 + it + \frac{t(t-1)i^2}{2!} + \frac{t(t-1)(t-2)i^3}{3!} + \ldots \qquad (1)$$

The first two terms of the expansion are equivalent to the simple interest assumption. Subsequent terms represent the compounding influence.

For all reasonable values of t and i, this function represents a telescoping series with each term smaller in absolute value than the one before. If t is an integer, the first (t+1) terms will be positive, and all subsequent terms will be zero. If t is not an integer, let T represent the truncated integer value of t. Then the first (T+2) terms will be positive, all subsequent terms will alternate in sign, and the cumulative value of the terms alternating in sign will be negative. Also, for all reasonable values of i and t, terms beyond the sixth have values less than .00001 and can be safely ignored.

The larger t is, the more consecutive terms in the expansion will be positive and, hence, the more simple interest will understate the true compound rate. Similarly, the larger i is (holding t constant), the greater will be the difference between simple and compound rates. Exhibit 1 shows annualized simple and compound yields at various bank discount rates and for different terms to maturity.

For $1/2 \leq t < 1$, which is the case for bank discount issues of more than one-half year to maturity, the cumulative value of all terms after the first two in Equation 1 will be negative, and thus simple interest will overstate the compound rate. This disparity grows at an increasing rate as i increases, or as t decreases (*i.e.*, as the maturity of the security increases). Exhibit 2 shows simple and compound annualized yields at various bank discount rates for securities with 240,

[2]This important point was suggested by a referee, who also brought to our attention a forthcoming article by Fielitz [2] which deals with issues similar to those covered in this paper.

Exhibit 1. Annualized Yields for Holding Periods Less Than One-Half Year

Annual Bank Discount Rate	10 Days		30 Days		90 Days		180 Days	
	Simple	Compound	Simple	Compound	Simple	Compound	Simple	Compound
1%	1.0145	1.0169	1.0149	1.0170	1.0164	1.0177	1.0190	1.0190
5	5.0769	5.1383	5.0907	5.1452	5.1336	5.1670	5.1994	5.2003
6	6.0937	6.1822	6.1139	6.1925	6.1760	6.2243	6.2715	6.2728
8	8.1294	8.2875	8.1656	8.3062	8.2766	8.3635	8.4491	8.4515
10	10.1674	10.4154	10.2242	10.4451	10.3989	10.5359	10.6725	10.6763
12	12.2077	12.5664	12.2896	12.6094	12.5430	12.7424	12.9433	12.9489
14	14.2503	14.7405	14.3621	14.8000	14.7093	14.9835	15.2628	15.2706
16	16.2951	16.9380	16.4416	17.0168	16.8981	17.2601	17.6329	17.6432
18	18.3417	19.1587	18.5279	19.2601	19.1099	19.5729	20.0549	20.0683
20	20.3911	21.4040	20.6215	21.5307	21.3450	21.9226	22.5309	22.5476

300, and 360 days to maturity. It also shows the coupon equivalent yield given by the Treasury Department's formula shown in Appendix B.

Implications

Examination of Exhibits 1 and 2 reveals a number of important points. For low bank discount rates, the differences among yields is negligible. This fact, coupled with the ease of calculation of bank discount yields, undoubtedly explains why this crude approximation has persisted for so long in spite of the existence of more exact methods.

For the few naive investors who think that bank discount is the actual yield, the Exhibits show that, in fact, bank discount badly understates the actual compounded yield at higher discount rates. For example, the yield on a 360 day Treasury bill is, even after adjusting for the impact of compounding, 189 basis points higher than the bank discount rate of 14 percent.

For maturities close to one-half year, the differences between simple and compound yields are slight. However, for short maturities and high bank discount rates, the differences are substantial and increase at an increasing rate as the bank discount rate rises or the maturity decreases. Such differences could conceivably change an investor's preference for bank discount issues versus coupon issues, even after realization that reinvestment might not be at the assumed rate. In the future, yields should be reported on a compound basis by issuers and financial publications in order to facilitate comparison with instruments of different maturities.

The coupon equivalent formula used by the Treasury Department is not a very good approximation of the compounding disadvantage inherent in discount issues with maturities greater than one-half year. More detailed tables than those provided here would show that the formula is good for maturities slightly more than six months or slightly less than one year, but is poor for the intermediate range (seven months to eleven months). Since with modern calculators it is easier to solve for the exact compound rate than it is to solve

Exhibit 2. Annualized Yields for Holding Periods in Excess of One-Half Year

Annual Bank Discount Rate	240 Days			300 Days			360 Days		
	Simple	Compound	Treasury*	Simple	Compound	Treasury	Simple	Compound	Treasury
1%	1.0207	1.0199	1.0195	1.0224	1.0207	1.0204	1.0241	1.0216	1.0216
5	5.2443	5.2229	5.2117	5.2899	5.2457	5.2362	5.3363	5.2688	5.2678
6	6.3368	6.3057	6.2894	6.4035	6.3391	6.3252	6.4716	6.3729	6.3715
8	8.5681	8.5116	8.4819	8.6905	8.5728	8.5474	8.8164	8.6352	8.6327
10	10.8631	10.7728	10.7253	11.0606	10.8716	10.8309	11.2654	10.9728	10.9688
12	13.2246	13.0916	13.0215	13.5185	13.2386	13.1784	13.8258	13.3901	13.3841
14	15.6557	15.4704	15.3726	16.0692	15.6772	15.5930	16.5052	15.8915	15.8832
16	18.1592	17.9115	17.7805	18.7180	18.1909	18.0780	19.3122	18.4823	18.4711
18	20.7386	20.4176	20.2475	21.4706	20.7836	20.6366	22.2561	21.1676	21.1530
20	23.3975	22.9915	22.7760	24.3333	23.4594	23.2726	25.3472	23.9536	23.9349

*The coupon equivalent yield using the Treasury Department's formula shown in Appendix B.

for the Treasury's approximation of that rate, it would seem prudent in the future to use the exact compounding formula shown in Appendix C for coupon equivalent yield calculations for instruments with maturities in excess of one-half year, as well as for shorter maturities.

Use of simple interest in place of compound interest badly overstates the yield on longer term bank discount instruments. However, this is the practice followed on the recently created All Savers Certificate. The yield on these instruments is set at 70 percent of the simple interest yield (referred to as the investment yield in financial publications) at the most recent auction of 52 week Treasury bills. This results in an overstatement of about 83 basis points at a 16 percent bank discount rate, and thus overstates the All Savers yield by about 58 basis points relative to a coupon issue of similar maturity.

Finally, the analysis presented here suggests a potential source of bias in a substantial number of articles by previous researchers. Many articles on term structure, risk premia, and cost of capital estimation using the CAPM framework contain no mention of coupon equivalent yields, leading to the conclusion that bank discount rates, rather than coupon equivalent yields, were probably employed, and that the impact of compounding was definitely not considered.[3] For example, Vandell and Stevens [7, p. 34] clearly used bank discount yields. Their Treasury bill yields are identical to values reported in the *Federal Reserve Bulletin* for new issues of 3 month bills. This oversight results in an average bias of 19 basis points relative to simple interest yields or 25 basis points relative to compound yields. The bias ranged from 13 basis points to 45 basis points (using the compounding method). Other recent articles in this and other respected journals contain a similar, but less explicit bias. This bias, while small in the past, will become quite large as the high interest rates of the 1979–1982 period are incorporated into future studies.

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[3]Notable exceptions are Ibbotson and Sinquefield [3], who used the holding period return calculation for one month Treasury bills, which would approximate the simple interest formula, and Carleton [1], who used compounding of short term rates (but for an annual, rather than six month period) in his analysis. While these articles did not compound to a semi-annual period, their methods were appropriate for their analyses, and could easily be adapted to the models presented in this paper.

Appendix A. Conversion of Bank Discount to Coupon Equivalent for Instruments with Less Than One-Half Year to Maturity

Let: i_b = the bank discount rate;
i_c = the coupon equivalent rate;
D = the number of days to maturity of the instrument;
Y = the number of days in a year (365 unless the security will be outstanding on a February 29th, in which case 366 is used);
P = the purchase price of the security.

Bank discount yield is calculated as:

$$i_b = \frac{100 - P}{100} \times \frac{360}{D}. \qquad \text{(A-1)}$$

To calculate the coupon equivalent yield, the purchase price is substituted for par value in the denominator of the first term, and Y is substituted for 360 in the numerator of the second term, giving:

$$i_c = \frac{100 - P}{P} \times \frac{Y}{D}. \qquad \text{(A-2)}$$

This is equivalent to simple interest calculations, but is called coupon equivalent by the Treasury Department and in financial publications.

Appendix B. Conversion of Bank Discount to Coupon Equivalent for Periods in Excess of One-Half Year

For discount issues with maturities in excess of six months, the coupon equivalent calculation shown in Appendix A overstates the actual yield because interest on the discount issue is not received until maturity, while interest on a coupon issue is received every six months. Accordingly, the Treasury Department uses the following formula to approximate the compounding effect:

In addition to the values defined in Appendix A, let:

S = number of days in one-half year (182.5 except for leap years);
N = number of days to maturity in excess of one-half year;
i_m = the coupon equivalent yield.

The following formula is then used to approximate the compounding effect:

$$1 = \left(1 + \frac{i_m}{2}\right)\left[1 + \left(\frac{N}{S}\right)\frac{i_m}{2}\right]\left(\frac{P}{100}\right). \qquad \text{(B-1)}$$

The Treasury solves this equation for i_m by an iterative procedure [6], using i_c as calculated in Appendix A as the upper limit and proceeding by trial and error until one estimate of i_m yields a value for Equation B-1 slightly greater than 1.0000 and a second estimate yields a value slightly less than 1.0000. Linear interpolation between the two estimates gives the final estimate of the coupon equivalent yield.

Since Equation B-1 is a quadratic equation in i_m, it can be solved directly [5]: Note that $(S+N) = D$, the number of days to maturity of the instrument. Then

$$i_m = \frac{\sqrt{B^2 - 4AC} - B}{2A}$$

where: $A = \dfrac{D}{2Y} - .25;$

$B = \dfrac{D}{Y};$

$C = \dfrac{P - 100}{P}.$

Appendix C. Compound Yield Formula

Using the variables defined in Appendices A and B, the actual compound yield, i_y, can be calculated as:

$$i_y = \left[\left(1 + \frac{(100-P)}{P}\right)^{S/D} - 1\right] \times 2. \qquad \text{(C-1)}$$

The rate is calculated for one-half year and multiplied by two, rather than compounding for the entire year, because coupon issues standardize at one-half year rather than a full year. That is, a bond paying \$60 interest semiannually is referred to as a 12 percent bond, not a 12.36 percent bond.